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S.Y.B.Sc. (NCF - NEP) (Sem. - IV) Examination September - 2025

Mathematics - MH-ME-401

Vector Calculus : Theory And Applications

Time: 2 Hour] **[Total Marks: 25**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
(2) All questions are compulsory
(3) Figures to the right indicate marks of the corresponding question.
(4) Follow usual notations and conversations.

Seat No.:

Student's Signature

Q. 1 Answer the following. (Any five) **05**

1. Define Vector Product of two vectors.
2. If $f = x^3 + y^3 + 3xyz$ then find grad f.
3. If $\vec{r} = t\hat{i} + (t^2 + 1)\,\hat{j} + (t^2 + t + 1)\,\hat{k}$, then find $\frac{d\vec{r}}{dt}$.
4. Define Divergence of a vector.
5. If $\vec{a} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} - 6\hat{k}$ then find $|\vec{a} + \vec{b}|$.
6. Define Harmonic function.

Q. 2 Answer the following. (Any Two) **10**

1. If $\vec{r} = (1 - \cos t)\,\hat{i} + (t - \sin t)\,\hat{j}$, then find $\frac{d\vec{r}}{dt}$, $\frac{d^2\vec{r}}{dt^2}$, $\left|\frac{d^2\vec{r}}{dt^2}\right|$ at t = 0.
2. If $\vec{a} = (5, -1, -3)$ and $\vec{b} = (1, 3, -5)$ then prove that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular to each other.
3. Find the angle between two vectors $\vec{a} = \hat{i} + \hat{j} - k$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

Q. 3 Answer the following. (Any Two) **10**

1. Find the directional derivative of $\phi = xyz$ in the direction of vector $\vec{a} = (3, 9, 3)$ at the point (3, 1, 3).
2. If ϕ is a differentiable scalar function and $\vec{V}$ is a differentiable vector function then prove that, $div\,(\phi\vec{V}) = (\text{grad}\,\phi)\,.\,\vec{V} + \phi\,(div\,\vec{V})$.
3. If $\vec{V} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$, then find $curl\,\vec{V}$.